High quality meshing with harmonic maps

E. Marchandise\textsuperscript{1}, J.F Remacle\textsuperscript{2}, C. Geuzaine\textsuperscript{3}

\textsuperscript{1} Institute of Mechanics, Materials and Civil Engineering (IMMC), emilie.marchandise@uclouvain.be
\textsuperscript{2} Institute of Mechanics, Materials and Civil Engineering (IMMC), jean-francois.remacl@uclouvain.be
\textsuperscript{3} Université de Liège, Department of Electrical Engineering and Computer Science, c.geuzaine@ulg.ac.be

Creating high quality meshes is an essential feature for obtaining accurate and efficient numerical solutions of partial differential equations as it impacts both the accuracy and the efficiency of the numerical method using those meshes.

In many cases, surfaces do not have a standard CAD representation and are only known by triangulations such as stereolithography (STL) triangulations. These kinds of surfaces are commonplace in many areas of science and engineering, e.g. in the form of 3D scanned images, terrain data, or medical data obtained from imaging techniques through a segmentation procedure. Such triangulations are often oversampled and/or of poor quality (with triangles exhibiting very small aspect ratios), which makes them unsuited for direct use by numerical methods like finite elements, finite volumes or boundary elements. This is also problematic for the volume mesh since the surface mesh serves as input for the volume meshing algorithms. Improving the mesh quality can then be performed using a remeshing procedure.

In the case of manufactured objects, the surfaces are often designed using a CAD system and described through a constructive solid geometry procedure. Non Uniform Rational B-Splines (NURBS) are commonly used for describing the shape of surfaces. NURBS surfaces are usually nice and smooth so that it is possible to produce high quality surfaces meshes using NURBS as input. However, most surface mesh algorithms mesh model faces individually, which means that points are generated on the bounding edges and that these points will be part of the surface mesh. If thin CAD patches exist in the model they will result in the creation of small distorted triangles with very small angles—even if the bounding edges of these thin patches have no physical significance. As in the case of a poor quality STL triangulation, a remeshing procedure is then also desirable.

In a recent paper \cite{1}, we have introduced an efficient approach for high quality remeshing of surfaces based on a parametrization technique (see Fig. 1). The approach uses a discrete finite element harmonic map to parametrize the input triangulation onto a unit disk. By combining it with a local cavity check algorithm that enforces the discrete harmonic map to be one-to-one, we came out with a robust method for remeshing that is advantageous compared with mesh adaptation methods. We show that, with the proposed approach, we are able to recover high quality meshes from both low input STL triangulations and complex surfaces defined by many CAD patches. We have also compared this approach with the one-to-one convex combination map of Floater \cite{4}.

We will discuss the two important limitations of harmonic maps, namely limitations on the genus and the geometrical aspect of the surface. Indeed, to be able to parametrize the triangulation onto a unit disk, the triangulation should be homeomorphic to a disk, i.e have a genus zero with at least one boundary. Besides, as the solution of harmonic maps tends exponentially to a constant, the triangulation should have an uniform geometrical aspect ratio to prevent non distinguishable coordinates.

We will present a robust and automatic way to overcome the topological and geometrical limitations
of harmonic maps (see Fig.2). The algorithm combines a recursive discrete harmonic mapping with a multi-level edge partitioning software that recursively partitions the triangulation into a small number of charts that satisfy the topological and geometrical constraints. We show that our method renders high quality meshes and highlight the benefits of using those high quality meshes for subsequent numerical simulations.

Figure 2: STL skull triangulations obtained from medical images (Left) that has been automatically remeshed with our automatic remeshing algorithm (Right).

The overall remeshing procedure is implemented, together with the finite element discretization procedure required for computing harmonic maps, in the open-source mesh generator gmsh [2].

References


