An Efficient Multi-Step Procedure For Enriching Limited Two-Dimensional Far-Field Pattern Measurements

H. Barucq¹, C. Bekkey², and R. Djellouli³

¹ LMA, CNRS UMR 5142, Université de Pau et des Pays de l’Adour & INRIA Futurs Research Center, Team-Project Magic3D, helene.barucq@univ-pau.fr
² Faculté des Sciences de Monastir, Tunisia, chokri.bekkey@ipeit.rnu.tn
³ Department of Mathematics, California State University Northridge & INRIA Futurs Research Center, Team-Project Magic3D, rabia.djellouli@csun.edu

Abstract

The determination of the shape of an obstacle from its effects on known acoustic or electromagnetic waves is an important problem in many technologies such as sonar, radar, geophysical exploration, medical imaging and nondestructive testing. This inverse obstacle problem (IOP) is difficult to solve, especially from a numerical viewpoint, because it is ill-posed and nonlinear [2]. Moreover, the precision in the reconstruction of the shape of an obstacle strongly depends on the quality of the given FFP measurements: the range of the measurements set and the level of noise in the data. Indeed, the numerical experiments performed in the resonance region (for example, see [4]–[6]) –that is, for a wavelength that is approximately equals to the diameter of the obstacle– tend to indicate that in practice, and at least for simple shapes, a unique and reasonably good solution of the IOP can be often computed using only one incident wave and full aperture far-field data (FFP given on the entire circle S), or using anywhere from 13 to 24 incident waves and limited aperture far-field data (FFP measured only at a limited range of angles), as long as the aperture is larger than π/2. For smaller apertures, the reconstruction of the shape of an obstacle becomes more difficult and nearly impossible for apertures smaller than π/4.

Given that, and the fact that from a mathematical viewpoint, the FFP can be determined on the entire S from its knowledge on a subset of S because it is an analytic function, we propose a solution methodology to extend the range of FFP data when measured in a limited aperture and not on the entire circle S. The ultimate objective is to use the enriched data to solve efficiently the IOP problem when only few FFP measurements are given. However, due to the analyticity nature of the FFP, the reconstruction or the extension of the far-field pattern from limited measurements is an inverse problem that is severely ill-posed, and therefore very challenging from a numerical viewpoint. Indeed, numerical results [3] indicate that the reconstruction of the FFP using the discrete L² minimization with the standard Tikhonov regularization is very sensitive to the the noise level in the data. The procedure is successful only when the range of measurements is given on an aperture larger than π/2, a case where there is no practical need for such extension [3].

Solution Methodology: we propose a multi-step procedure for extending/reconstructing the FFP from the knowledge of limited measurements in order to solve inverse acoustic problems, when only limited aperture data of the far field pattern is available. The proposed solution methodology addresses the ill-posedness nature of this inverse problem using the total variation of the FFP coefficients as a penalty term that involves the L¹ norm. Consequently, the new cost function is no longer differentiable. We restore the differentiability to the cost function using a perturbation technique [5], which allows us to apply the Newton algorithm to minimize the resulting non-linear but differentiable cost function.

We have investigated the effect of the number of measurements, the frequency regime, and the noise level on the performance of the proposed solution methodology [1]. Numerical results obtained in the case of two-dimensional sound-soft disk-shaped scatterer will be presented. These results illustrate the potential of the solution methodology for enriching the FFP data. In particular, we will
show that the method is successful in enriching the backscattering measurements, the most realistic situation for practitioners, to an aperture of 90 degrees, in the low frequency regime with a relative error less than 10% [1].

References


