Unified Geometric Model of Elastic Rods and Viscous Threads

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A discrete treatment of adapted framed curves, parallel transport, and holonomy is developed, thereby establishing the language for a discrete geometric model of thin flexible rods and viscous fluid threads [1, 2]. The model captures the stretching, bending, and twisting modes of slender materials well-described by a centerline curve, and it accommodates arbitrary materials cross sectional shape and (curved) undeformed configurations. A manifold projection method for coupling rods to rigid-bodies is briefly discussed.

This kinematic description represents the orientation of the material frame by its angular deviation relative to a reference frame. Examination reveals two natural choices for the reference frame, corresponding to parallel transport along (a) the spatial (centerline) dimension or (b) the temporal dimension. The former, when applied to isotropic cross sections and straight undeformed configurations, and combined with a quasistatic treatment of the material frame, leads to a collapse of the formulation into a special, simple form with significantly reduced computation. By contrast, for the general case of arbitrary cross sections and/or curved undeformed configurations, the time-parallel reference frame is demonstrated to be preferrable, as it leads to a banded stiffness matrix, making economical the computation of implicit time integration methods.

The model is validated via quantitative buckling, stability, and coupled-mode experiments; applications are presented for rods problems such as knot-tying and plectoneme formation, and for viscous fluid thread problems such as coiling and meandering on a moving belt. Application examples compared against theoretical and experimental results.

Figure 1: Plectonemes in twisted rods, and coils in poured viscous fluids, as captured by the model.

References