Optimal Shape Design of Shell Structures

M. Shimoda\textsuperscript{1}, K. Iwasa\textsuperscript{2}, S. Tsukada\textsuperscript{3}

\textsuperscript{1}Shonan Institute of Technology, Japan, shimoda@md.shonan-it.ac.jp
\textsuperscript{2}Kobelco Cranes Co., Ltd., Japan, iwasa_kyohei@kobelconet.com
\textsuperscript{3}Graduate School of Shonan Institute of Technology, Japan

Shell structures are extensively used in all fields of engineering. Examples include architectural structures, hydraulic structures, containers, automobiles, spacecraft, aircraft, ships, machine parts, instruments, etc. We can find them in nature such as an egg, sea-shells, insects. Although shell structures are thin and light, they span over large areas, and have high load-carrying capacity. In the design of a shell structure, it is important to optimize its shape in order to make the structure light with the minimum amount of material, satisfying the various mechanical characteristics, functions and sometimes beautiful form.

In this paper, we present a numerical optimization method for the optimal shape design of shell structures. As the design variables to determine the shape of a shell structure, one can consider the in-plane shape variation that moves in the tangential direction to the surface and/or the out-of-plane shape variation that moves in the normal direction to the surface. In our previous studies, we proposed shape optimization methods for both the in-plane shape variation in order to maintain the curvatures of the given initial shape \cite{1} and the out-of-plane shape variation in order to determine the optimal free-form shell \cite{2}. In this paper, we will apply the shape optimization method for the out-of-plane shape variation to a stiffness design problem and a vibration eigenvalue design problem of shell structures. Using the compliance as an objective functional in the stiffness design problem, the compliance is minimized subjected to a volume and the state equation constraints. In the vibration eigenvalue design problem, the vibration eigenvalue is maximized under the same constraint conditions as the stiffness design problem. Each design problem is formulated as a distributed-parameter, or non-parametric, shape optimization problem under the assumptions that the shell is varied in the normal direction to the surface and the thickness is constant, and then the shape gradient function and the optimality conditions are theoretically derived using the material derivative method, the Lagrange multiplier method and the adjoint variable method. The optimum shape is determined by applying the shape gradient function to the Robin-type traction method \cite{2}. The Robin-type traction method was developed for the shell optimization with the out-of-plane shape variation based on the original traction method. The traction method, which is a gradient method in the Hilbert space and has been developed as a non-parametric shape optimization method by the authors. In the traction method, the negative shape gradient function is applied in the normal direction to the design surface as an external force to vary the shape. The resultant displacement field which represents the amount of domain variation (i.e. design velocity field) is added to the original shape. Using this method, the smooth domain variation that minimizes the objective functional can be obtained. In the shape variation analysis, or the velocity analysis in the Robin-type method, earth springs were added to design surfaces. Other advantages offered by this method are summarized as follows: (1) it is not needed to parameterize the shape unlike the basis vector method, because all nodes on the design domain can be moved as the design variable, (2) It is not necessary to refine the mesh. (3) It can be easily implemented combining with a standard FEM code. For simplicity, a shell structure is discretized by triangular planar shell elements with the Mindlin-Reissner plate theory in FE analysis for calculating the objective functional and the shape gradient function.

The method is applied to several stiffness and vibration design problems. The volume constraint is
set as 1.05 times the initial value in all problems. Figure 1 shows the optimization result of the stiffness design problem of a U-bracket under a bending load. The initial shape with the boundary condition in the stiffness analysis and the obtained result with the shape constraint condition in the velocity analysis are shown in Fig. 1-(a) and (b). In the velocity analysis, the restraint condition is set independent from that in the stiffness analysis considering the shape design condition. In Fig. 1-(b), some beads were efficiently created, and the compliance was reduced by about 87% while satisfying the given volume constraint. Figure 2 shows the obtained result of the stiffness design problem of a clamped square plate under a snow load. A wavering shape was created while reducing the compliance by about 98%. Figure 3 shows the optimization result of the vibration eigenvalue design problem of a rectangular plate which is simply supported at both ends. The 1st eigenvalue of the initial shape was increased by about 88 times the initial shape tracking the 1st eigenmode. Figure 4 shows the obtained result of the vibration design problem of a roof. The 1st eigenvalue of the initial shape was increased by 10 times the initial shape tracking the 1st eigenmode. The optimal shape with beads in the top and side surfaces was obtained.

The validity and practical utility of this method for the optimal free-form design of shell structures with the out-of-plane shape variation were verified from the obtained results. By using this method, the optimal curvature distribution can be determined without shape parameterization. We confirmed the initial bending-carrying structures were changed to the membrane-carrying ones by this method.

Figure 1: Optimization of U-bracket in stiffness design problem

(a) Initial
(b) Optimal

Figure 2: Optimization of square plate in stiffness design problem

(a) Initial
(b) Optimal

Figure 3: Optimization of rectangular plate in eigenvalue design problem (1st mode tracked)

(a) Initial
(b) Optimal

Figure 4: Optimization of roof in eigenvalue design problem (1st mode tracked)

(a) Initial
(b) Optimal

References