Displacement Extrapolation Method : An alternative to J-integral for stress intensity factors computation using X-FEM

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Abstract:

1 Introduction

Stress intensity factors (SIFs) computation techniques based on J-integral methods [1] are widely used in finite element codes. However, the Displacement Extrapolation Method (DEM) [2] is an alternative way to compute them. The idea is to process an extrapolation based on the relative displacement of the crack face nodes. This method is generally less accurate than integral methods, but it requires much less computational resources. The method is well documented and is implemented in some finite element software. The X-FEM introduced in [3] is suited for crack problems. However, the literature on DEM in the X-FEM case seems to be non-existent in the present time. This paper tries to fill this lack in the literature by describing a new method using X-FEM. The approach followed here is only developed for Linear Elastic Fracture Mechanics. Note that the finite element software Code_Aster [4] already computes SIFs with DEM in X-FEM, but in a slightly different way.

2 Stress Intensity Factors - Distant Factors

If we define by $U_x, U_y, U_z$ the relative displacement of the crack faces (with axis local to the crack front), the SIFs are defined as follows:

$$K_I = \lim_{r \to 0} \frac{\mu U_y}{\kappa + 1} \sqrt{\frac{2\pi}{r}}$$

$$K_{II} = \lim_{r \to 0} \frac{\mu U_x}{\kappa + 1} \sqrt{\frac{2\pi}{r}}$$

$$K_{III} = \lim_{r \to 0} \mu U_z \sqrt{\frac{\pi}{2r}}$$

The three corresponding modes are shown in Fig. 1. It is possible to have an approximation of the SIFs by extrapolating the relative displacements near the crack tip. Let’s define the distant factor $K_I^*(r)$ (resp. $K_{II}^*(r), K_{III}^*(r)$) obtained by ignoring the limit in the definition of $K_I$ (resp. $K_{II}, K_{III}$). For mode I:

$$K_I^*(r) = \frac{\mu U_y(r)}{\kappa + 1} \sqrt{\frac{2\pi}{r}}$$

Using [5] and [6], it is possible to show that in mode I, by extending the asymptotic solution to higher orders:

$$K_I^*(r) = K_I - \frac{4A_2 r}{3} \sqrt{\frac{\pi}{2}} + ...$$

where $A_2$ depends on the loading and on the geometry. This relation shows that $K_I^*(r)$ should vary linearly in the vicinity of the crack front.
3 Displacement Extrapolation Method in the X-FEM framework

In X-FEM, the crack is usually not meshed. It means that some post-processing is needed to obtain the relative displacement of the crack faces. The approach followed here is based on an underlying 2D mesh of the crack faces generated by cutting the elements from the 3D mesh. The relative displacement of the faces are computed on the nodes of this mesh.

One of the studied cases is a simple cracked cube loaded in traction. The corresponding 3D mesh is shown in Fig. 2. The parameter \( n \) defines the discretization of the side of the cube. Numerically, we observed that near the crack front, \( K_I^* (r) \) varies almost linearly except in a tiny zone extremely close to the front which size is related to \( h \) (the characteristic length of the mesh near the crack front), due to the presence of the singularity. This is shown in Fig. 3 for \( n = 10 \).

![Figure 2: Mesh of the cube (n = 10), the position of the crack is highlighted in bold.](image)

For each desired crack front position, the SIFs are extrapolated from the relative displacements of a set of nodes in a neighbourhood of the crack front. This set can be defined as in Fig. 4. The extrapolation consists of a linear regression using a generalised least square method. The extrapolated \( K_I \) is given by the \( r = 0 \) value of the dashed line in Fig. 3.

![Figure 3: \( K_I^* (r) \) (n = 10, h \approx 0.1) (image)]

For each case studied, the results are compared with the SIFs computed from the J-Integral for the same case. The accuracy of the method strongly depends on the mesh refinement. Provided the mesh is fine enough, especially in a thin area surrounding the crack faces, the relative difference between the results of the two methods remains below 1 % in case of mode I loading. For mixed mode loading, this difference may rise to 5%. In some cases, depending on the size of the area used to extrapolate, the extrapolated SIFs values are even smoother (along crack front) than the ones computed with J-integral.

4 Conclusion

DEM and domain integral methods give very similar results. DEM is much lighter because it does not need complex computation (Eshelby tensor, virtual crack advance function...). The main advantage of the method is that the extrapolation only requires displacements on the crack face and does not require a huge refinement out of this surface. However, the method supposes that the crack front is smooth enough and that the crack faces are plane enough too.

References