Transmission Eigenvalues and their Application to Inverse Scattering Problems

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The interior transmission problem arises in inverse scattering theory for inhomogeneous media. It is a boundary value problem for a set of equations defined in a bounded domain coinciding with the support of the scattering object. Of particular interest is the spectrum associated with this boundary value problem, more specifically the existence of eigenvalues known as transmission eigenvalues. Indeed, on one hand, in the context of sampling methods for reconstructing the support of the scatterer one needs to avoid those frequencies that correspond to transmission eigenvalues, and hence it is important to know that the transmission eigenvalues form a discrete set. On the other hand, one can use transmission eigenvalues to obtain information about physical properties of the scattering medium [1], [3] and therefore it is important to know whether they exist and to understand their connection with the index of refraction. The latter application is based on the recent results in [2] which justify the numerical observation that the transmission eigenvalues can be computed from the far field data. Either way, the investigation of the spectral properties of the interior transmission problem has become an interesting mathematical question in inverse scattering theory.

We present here the most recent developments on transmission eigenvalues, in particular we show the existence of infinitely many transmission eigenvalues and provide lower and upper bounds for the first transmission eigenvalue in terms of the geometry and physical properties of the scattering object [4], [5]. Then, we show how to use these bounds to obtain information on the index of refraction of a general anisotropic scattering medium, as well on the presence of defects inside the scattering medium.

References