Static Shakedown Analysis of Multi-layered Pavements

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Based on an analytical solution [1] to shakedown problem of a half space subjected to moving surface loads, a numerical static approach, together with a finite element implementation in ABAQUS via user-interface UVARM, is developed for shakedown analysis of multi-layered pavements under moving traffic loads and a lower bound to shakedown limit is obtained. The proposed numerical approach is further validated by experimental tests for the Mohr-Coulomb type materials and can provide a powerful tool for stability analysis and engineering design of pavements.

Methodology, results and discussions

In 2005, Yu [1] presented an analytical solution to shakedown problem of a half space subjected to moving Hertz-distribution surface loads $P$. The Mohr-Coulomb yield criterion is assumed for cohesive-frictional materials and the Melan static shakedown theorem [2] is used. Therefore a lower bound to shakedown limit can be predicted, as formulated in Eq. (1) in terms of the material properties (cohesion $c$ and friction angle $\phi$) and the elastic stresses of the corresponding fixed Hertz load [3]. The procedure of applying Eq. (1) is to find the optimal minimum shakedown multiplier $\lambda_{sd}$ through the whole structure with $\lambda_{sd} \cdot P$ being the shakedown limit. The equation (1) is proposed for a half-space under moving loads and can be used for shakedown analysis of one-layered pavement.

$$\lambda_{sd} \leq \frac{c}{\sqrt{(\sigma_{zz}^e)^2 + \sigma_{zz}^e (\tan \phi)}}$$

(1)

However, in practice, most of pavements used in civil engineering are multi-layered. In order to extend and apply the analytical solution (1) to shakedown analysis of a multi-layered pavement, a numerical approach combining the elastic analysis of multi-layered soil pavements with investigation of the critical state (1) through all layers of pavements is developed in this paper. The optimal shakedown multiplier can be found over the whole structure. Various loading types can be evaluated and the effect of material parameters on the shakedown limit can be further investigated. The developed numerical method is linked into the commercial finite element (FE) software ABAQUS via user-interface UVARM so that the FE implementation of the proposed approach can be achieved. Both 2-dimensional and 3-dimensional analyses are presented and compared.

First, in order to validate the proposed numerical approach, a single layered pavement is simulated and the numerical result has a very good agreement with the analytical solution [1], as shown in Figure 1, where $p_{max}$ is the maximum peak compressive pressure of the Hertz load. In the numerical analysis, the size of the simulated region of the pavement is selected to be sufficiently large so that a half-space requirement is approximately satisfied. Moreover, the mesh has to be fine enough, especially in the vicinity of the loading area, to obtain the elastic stresses in a reasonable accuracy.

To further validate the developed numerical approach, more simulations are implemented for a multi-layered pavement and compared with experimental tests by Juspi [4] in NCG lab in University
of Nottingham. In the experiment, the soil specimens (Granite-Portaway Sand) were compacted into moulds and tested in a slab test facility to observe the shakedown phenomenon of soils under moving wheels. Various deformation development curves were plotted due to different load levels and the corresponding modes were classified as: Type 1 (below shakedown); Type 2 (intermediate shakedown) and Type 3 (non-shakedown). In the experiment, the contact area grows with the increase of applied loads, while in the numerical simulations, an averaged contact area is adopted. The numerical results are presented in Table 1, which can well predict the shakedown status of multi-layered pavement.

More numerical analyses of a two-layered pavement are implemented, where the first layer has the Mohr-Coulomb material with the poisson ratio 0.2 and the second layer has the Tresca material with the poisson ratio 0.49. The numerical results about the influences of the strength ratio $c_1/c_2$, the stiffness ratio $E_1/E_2$, and the friction angle of the first layer $\phi_1$ are presented in Figures 2 and 3. It can be seen that the dimensionless shakedown limit increases with the rise of the strength ratio but has a limit due to the transfer of the failure point from the first layer to the second layer. It also implies that there exists an optimum stiffness ratio for the engineering design of multi-layered pavements.

![Experiment vs FEM results](image1)

<table>
<thead>
<tr>
<th>Contact area (mm²)</th>
<th>6570</th>
<th>8856</th>
<th>7713</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{max}$ (kPa)</td>
<td>229*</td>
<td>339**</td>
<td>276</td>
</tr>
</tbody>
</table>

* maximum wheel pressure with type 1 response.
** minimum wheel pressure with type 2 response.

![Numerical with analytical results](image2)

![Influence of $c_1/c_2$ on shakedown](image3)

![Influence of $E_1/E_2$ on shakedown](image4)

**References**


