Higher-order continuum for wave dispersion in microstructured membranes

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The interest in tensioned membrane structures is of importance due to their practical application in many and various fields, such as architecture and structural engineering, bio-technology and acoustics [4]. In order to provide these high performance systems with the characteristics of flexibility and low weight, the use of appropriate composite materials is becoming fundamental. In mechanical problems involving materials with microstructure, the existence of microscopic internal characteristic lengths, introduces a size dependence into the structural response on the macroscopic level. Among other observed phenomena that are influenced by the microstructural characteristics, the dispersion of waves is of great interest in the mechanics of materials because it plays a crucial role for instance in stopping the formation of shock waves into systems. Dispersion is the phenomenon extensively documented by experimentalists that waves with different wave lengths propagate with different velocities [2, 3, 6]. Consequently, accurate modelling of such structures and materials requires that the microstructure is accounted for.

This contribution addresses the problem of elastic wave dispersion in composite membranes that possess a microscopic structure. As an alternative to the direct analysis of the discrete material that at microscopic level has been assumed here as a periodic elastic lattice (see Figure 1), continualisation techniques can be used to translate the discrete nature into a gradient-enriched continuum with the aim to preserve the essential link between the degrees of freedom of the actual model and those of the continuum [5]. The strategy of replacing such a discrete model with an equivalent continuum offers a efficient alternative since the continuum mechanics approach is computationally less expensive. A gradient-enriched model is obtained to account for the characteristic microstructural length scale and to predict the dispersion of the wave born out of heterogeneity of the material.

Figure 1: The two-dimensional lattice is assumed to vibrate in direction perpendicular to its planar position.
In the first part of this study, continuum models linked with the underlying material have been introduced taking into account the discreteness of the microstructure [1, 7]. From the application of different continualisation approaches, it has been shown that gradient-enriched continuum theories based on the use of Padé approximants introduce higher-order inertia terms as well as higher-order stiffness terms in classical models and improve the continuum description by including the length parameter characterizing the heterogeneity. The analysis of closed-form dispersion relations demonstrated that the enhanced continuum models are able to capture the physical phenomena of wave dispersion which is overlooked by classical continuum theories and also do not generate numerical instability.

In the second part, the field equations have been discretized in space by means of simple four noded finite elements. Importantly, the higher-order terms do not lead to an increase in the system size with respect to the classical elasticity. The evaluation of the convergence rate confirms the appropriateness of the adopted finite element discretisation. Finally, the numerical analysis of a microstructured membrane has been conducted. In order to illustrate the dispersion of the elastic waves, the response of the membrane to impulsive point load has been simulated (see Figure 2). Numerical results are in agreement with the analytical dispersion analysis presented in the first part.

References


