A modal derivative approach for model reduction in MEMS nonlinear dynamic analysis

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1 Introduction

Practical MEMS applications feature non-linear effects that are important to be realistically simulated. This typically involves large dynamic non-linear finite element (FE) models, and therefore efficient model reduction techniques are of great need. A projection basis constituted by few vibration modes enriched with modal derivatives (MDs) [1] can describe the main effect of nonlinearity without the need of a full model solution, as required by Proper Orthogonal Decomposition (POD). This approach showed promising results for a simplified nonlinear coupled problem, where the electrostatic forces were modelled with one-dimensional elements [2]. We present the application of MDs on a realistic three-dimensional MEMS example featuring geometrical nonlinear kinematic, squeeze-film damping effect, and explicit discretization of the electrostatic field.

2 Approach

Once discretized in space with the Finite Element method, the coupled problem is governed by the following system of equations [3]:

\[ \begin{align*}
M_{uu} \ddot{u} + f_m(u) - f_e(u) - f_p(u) &= 0 \\
K_{vv} v - q_{ext}(V, u) &= 0 \\
R(p)u + L(u)p + D(u, p)p &= 0
\end{align*} \]

(1)

where the first, second and third line refers to the structural, electrostatic and fluid problem, respectively. The unknowns of the problem \( u, v \) and \( p \) are the structural, electrical and pressure degrees of freedom, respectively. The electrostatic, fluid and mechanical forces are indicated with \( f_e, f_p \) and \( f_m \). \( M_{uu} \) is the structural mass matrix, \( q_{ext} \) and \( K_{vv} \) are the external charges and the electrical stiffness matrix, respectively. \( V \) is the applied voltage, while \( L, R \) and \( D \) are the matrices of the squeeze-film damping equation. The dependency of the various terms on the unknowns of the problem is indicated in 1.

As in [2], we propose a reduction basis for the structural problem:

\[ u = \Psi q \]

(2)

formed by \( K \) vibration modes \( \Phi_i \) calculated around a certain static equilibrium configuration and
their corresponding derivatives with respect to the modal amplitudes \( \frac{\partial \Phi_i}{\partial q_j} = \Phi_{ij} \), as

\[
\Psi = [\Phi_i \Phi_{ij}], \ i, j = 1, \ldots, K \tag{3}
\]

The modal derivatives \( \Phi_{ij} \) can be calculated, for instance, by differentiating the eigenvalue problem obtained by a static condensation the electrical and pressure degrees of freedom [2]. Once the structural basis \( \Phi \) has been calculated, a similar approach can be applied to the electrostatic equation by differentiation along the structural modal directions to obtain a reduced basis for the nodal voltages \( v \). A reduction basis for the pressure \( p \) can be readily obtained by applying the structural basis as imposed velocity field on the third set (fluid equation) of 1 around the chosen equilibrium configuration.

3 Preliminary results

The MD approach proved to be a promising method for two-dimensional micro-beam electromechanical analysis [2], as described in Figure 1. In our contribution, we will extend the approach to a fully coupled three-dimensional electromechanical problem including squeeze film damping.

![Figure 1](image1.png)

Figure 1: Comparison between full model response, POD based and MD based reduction of a micro beam subjected to step voltage [2]. The POD basis is obtained by sampling the results of a full nonlinear dynamic solution in the case of a 50 V step load. The MD basis is calculated around the static equilibrium configuration corresponding to the same voltage level. The two reduction basis are of the same size for a fair comparison. The MD basis is able to reproduce the dynamic behavior for a broad range of voltage levels up to dynamic pull-in.

References

